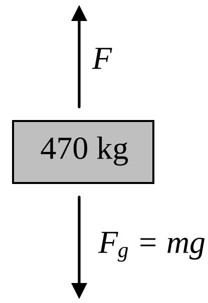
UNIT 1 REVIEW - SOLUTIONS RTD EXAM PREP

1. a) Uniform motion (balanced forces)



$$|\vec{F}| = |\vec{F}_g|$$

$$F = m\,g$$

$$= (470\text{ kg})(9.81\text{ N/kg}) = 4610.7\text{ N}$$

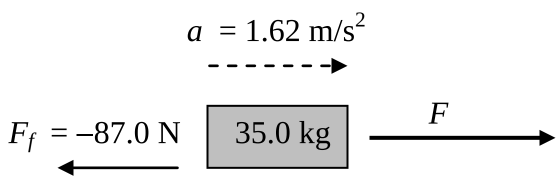
$$= 4.61\text{ kN}$$

b) $v = \dfrac{d}{t}$ $\quad d = v\,t$

$$= (2.80\text{ m/s})(5.50\text{ s}) = 15.4\text{ m}$$

2. a) Uniform acceleration (unbalanced forces)

Ref: Right +, Left –



$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F} + \vec{F}_f = m\vec{a}$$

$$\vec{F} + (-87) = (35)(+1.62)$$

$$\vec{F} = 143.7\text{ N}$$

$$= 144\text{ N right}$$

b) $\vec{v}_i = 0$; $\vec{a} = 1.62\ \text{m/s}^2$; $\vec{d} = 4.40\text{ m}$; t ?

$$\vec{d} = \vec{v}_i\,t + \frac{1}{2}\vec{a}\,t^2 \qquad d = 0.5a\,t^2 \qquad t^2 = \frac{d}{0.5a}$$

$$t = \sqrt{\frac{d}{0.5a}} = \sqrt{\frac{4.4}{0.5(1.62)}} = 2.33\text{ s}$$

3. $m = 1400$ kg ; $\vec{v}_i = 36 \times \dfrac{1000\ m}{3600\ s}$ = 10 m/s

Ref: East +, West –

$\vec{I} = -7.50 \times 10^3\ N \cdot s$; $\vec{v}_f$?

$\vec{I} = \vec{F}\,\Delta t = m\,\Delta\vec{v}$ $\quad\quad$ $\Delta\vec{v} = \dfrac{\vec{I}}{m} = \dfrac{-7.50 \times 10^3}{1400} = -5.357$ m/s

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ $\quad\quad$ $\vec{v}_f = \Delta\vec{v} + \vec{v}_i$

= (–5.357 m/s) + (10 m/s) = 4.64 m/s East

4. $m = 0.620$ kg ; $\vec{v}_i$ = –18.4 m/s ; $\vec{v}_f$ = 11.7 m/s

Ref: Up +, Down –

$t = 330 \times 10^{-3}$ s = 0.330 s

a) $E_{k\,i} = \dfrac{1}{2} m v_i^2$ = 0.5 (0.620 kg) (18.4 m/s)^2 = 104.9536 J

$E_{k\,f} = \dfrac{1}{2} m v_f^2$ = 0.5 (0.620 kg) (11.7 m/s)^2 = 42.4359 J

$\Delta E_k = E_{k\,f} - E_{k\,i}$ = (42.4359 J) – (104.9536 J) = –62.5 J

Thus, the ball lost 62.5 J of kinetic energy.

b) Method 1: $\quad \vec{F}\,\Delta t = m\,\Delta\vec{v}$

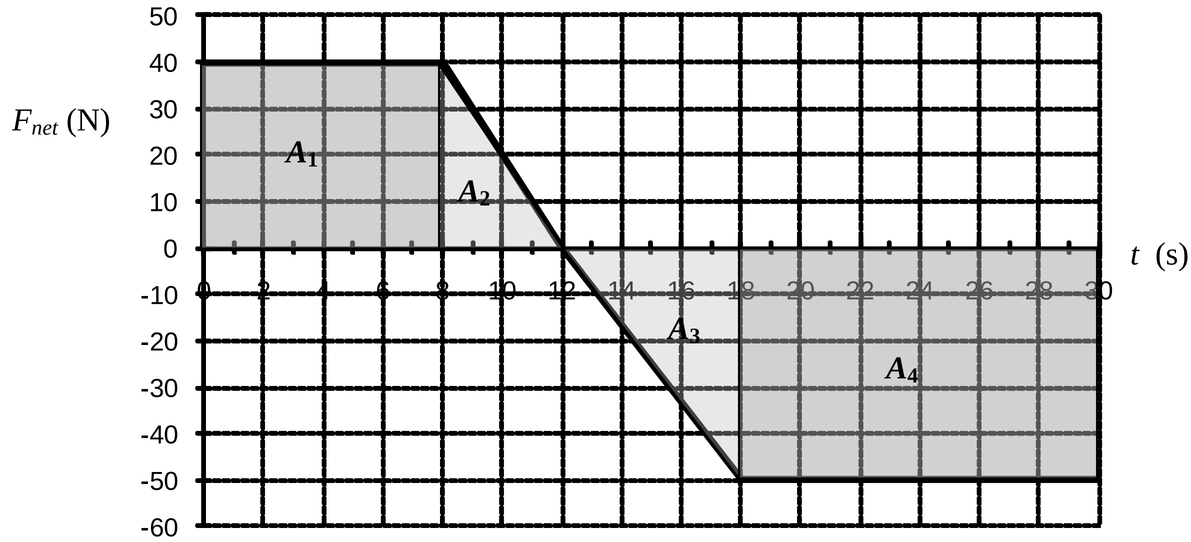
$$\vec{F} = \frac{m\,\Delta\vec{v}}{\Delta t} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = \frac{0.620\,(-18.4 - 11.7)}{0.330} = -56.6\text{ N} = 56.6\text{ N West}$$

Method 2:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{-18.4 - 11.7}{0.330} = -91.212\text{ m/s}^2$$

$\vec{F}_{net} = m\,\vec{a}$ = 0.62 kg (–91.212 m/s^2) = –56.6 N = 56.6 N West

5.



a) Impulse = total area for the first 12 seconds

= $A_1 + A_2$

= (8 s) (40 N) + 0.5 (4 s) (40 N) = 400 N·s

Since $\vec{v}_i = 0$ (starts at rest),

$\vec{I} = \vec{F}\,\Delta t = m\,\Delta\vec{v}$ $\quad\quad$ $\vec{I} = m\left(\vec{v}_f - \vec{v}_i\right)$ $\quad\quad$ $I = m\,v_f$

$$m = \frac{I}{v_f} = \frac{400}{7.75} = 52\text{ kg}$$

b) Impulse = total area for the entire 30 seconds

= $A_1 + A_2 + A_3 + A_4$

= (8 s) (40 N) + 0.5 (4 s) (40 N) + 0.5 (6 s) (–50 N) + (12 s) (–50 N)

= –350 N·s

$\vec{I} = m\,\Delta\vec{v}$ $\quad\quad$ $\vec{I} = \Delta\vec{p}$

Thus, the impulse is the change in momentum of the object.

$\left|\Delta\vec{p}\right| = 350$ N·s

6. Impulse ($\vec{I} = m\,\Delta\vec{v}$) is not affected by cushioning. The impulse is the same whether there is an airbag or not.

   Based on the equation $\vec{F}\,\Delta t = m\,\Delta\vec{v}$:

   $$F = \frac{m\,\Delta v}{\Delta t} \qquad\qquad F \propto \frac{1}{\Delta t}$$ Inverse relationship.

   The landing mat increases the time to change the stunt person's momentum, and thus, it decreases the force on the person.

7. a) If momentum is conserved, then the total momentum remains constant.

   $$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

   Thus, if the total momentum before a collision is 50 kgm/s right, then the total momentum after a collision is **50 kgm/s right**.

   RTD EXAM PREP

   b) If momentum is conserved, then the impulse on the objects are equal but opposite (based on Newton's 3rd Law). Since impulse is change in momentum, it follows that the change in momentum for each object is equal but opposite.

   Thus, if the change in momentum of piece 1 is 100 kgm/s North, then the change in momentum of piece 2 is **100 kgm/s South**.

   c) If momentum is conserved, then the impulse on the objects are equal but opposite (based on Newton's 3rd Law).

   Thus, if the impulse on piece 1 during an explosion is 400 Ns upward, then the impulse on piece 2 during an explosion is **400 Ns downward**.

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8. a) Conservation of momentum

Ref: Right +, Left –

This assumes that the system is isolated
(i.e. the net force on the entire system is zero).

$$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = m_1\,\vec{v}_{1f} + m_2\,\vec{v}_{2f}$$

$$(0.25)\,(6.0) + (0.37)\,(-10) = (0.25)\,(-4.0) + (0.37)\,v$$

$$-2.2 = -1.0 + 0.37\,v$$

$$v = \frac{-1.2}{0.37} = -3.243 \text{ m/s} = 3.2 \text{ m/s left}$$

b) To be elastic, total kinetic energy must remain conserved (constant).

$$E_{kTi} = E_{k1i} + E_{k2i} = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$$

$$= 0.5\,(0.25)\,(6)^2 + 0.5\,(0.37)\,(10)^2 = 23 \text{ J}$$

$$E_{kTf} = E_{k1f} + E_{k2f} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$= 0.5\,(0.25)\,(4)^2 + 0.5\,(0.37)\,(3.243)^2 = 3.9 \text{ J}$$

Since $E_{kTi} \neq E_{kTf}$, this collision is not elastic.

About 19 J was lost due to heat / sound.

9. Conservation of momentum

Ref: North +

$$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{12\,f}$$

$$m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = m_{12}\,\vec{v}_f$$

$$M(12) + 0 = (M + 4M)\,v$$

$$12M = 5M\;v$$

$$v = \frac{12M}{5M} \quad = \quad 2.4 \text{ m/s}$$

10. Since the system starts at rest, the total momentum at the start is zero.
Thus, the total momentum at the end must be zero as well (total remains constant)

Conservation of momentum (cannon = 1; ball = 2)

Ref: Forward +, Backward –

$$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

$$0 = \vec{p}_{1\,f} + \vec{p}_{2\,f}$$

$$0 = m_1\,\vec{v}_{1\,f} + m_2\,\vec{v}_{2\,f}$$

$$m_1\,\vec{v}_{1\,f} = -m_2\,\vec{v}_{2\,f}$$

$$\vec{v}_{1\,f} = -\frac{m_2\,\vec{v}_{2\,f}}{m_1} \quad = \quad -\frac{(3.90)(220)}{144 - 3.90} \quad = \quad -\frac{(3.90)(220)}{140.1}$$

$$= \quad -6.12 \text{ m/s} \quad = \quad 6.12 \text{ m/s backwards}$$

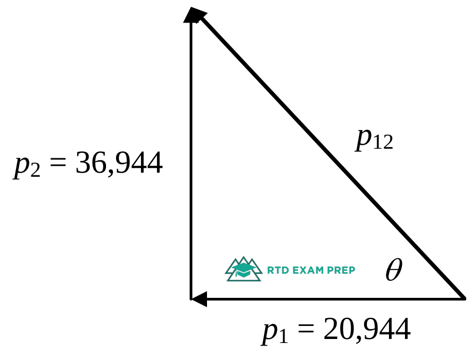
11. $v_1 = 58 \times \dfrac{1000\ m}{3600\ s} = 16.11$ m/s ; $p_1 = m_1 v_1$ = (1300 kg) (16.11 m/s) = 20,944 kg·m/s (West)

$v_1 = 70 \times \dfrac{1000\ m}{3600\ s} = 19.44$ m/s ; $p_2 = m_2 v_2$ = (1900 kg) (19.44 m/s) = 36,944 kg·m/s (North)

Conservation of momentum

$$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_{12}$$



Solving the triangle,

$$p_{12} = \sqrt{20{,}944^2 + 36{,}944^2} \;=\; 42{,}468 \text{ kg·m/s}$$

$$p = mv \qquad v = \frac{p_{12}}{m_{12}} \;=\; \frac{42{,}468}{1300 + 1900} \;=\; 13.3 \text{ m/s (or 47.8 km/h)}$$

$$\tan\theta = \frac{36{,}944}{20{,}944} \qquad \theta = \tan^{-1}(1.764) \;=\; 60.5° \text{ N of W (or 29.5° W of N)}$$

12. Method 1: Using a vector sum triangle

Before the explosion, it is at rest (p = 0)

After the explosion: $p_1 = m_1 v_1$ = (2 kg) (72 m/s) = 144 kg·m/s (North)

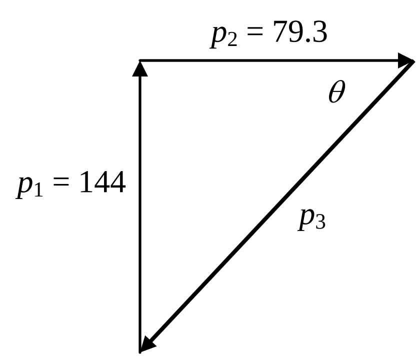
$p_2 = m_2 v_2$ = (1.3 kg) (61 m/s) = 79.3 kg·m/s (East)

Conservation of momentum

$$\vec{p}_{T\,i} = \vec{p}_{T\,f}$$

$$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

This means that if you add the three vectors together, you get a zero resultant (i.e. it returns to the start)



Solving the triangle,

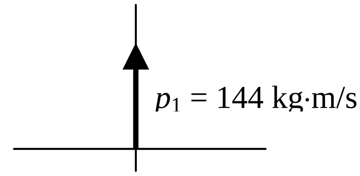
$$p_3 = \sqrt{144^2 + 79.3^2} \;=\; 164 \text{ kg·m/s}$$

$$\tan\theta = \frac{144}{79.3} \qquad \theta = \tan^{-1}(1.816) \;=\; 61.2° \text{ S of W (or 28.8° W of S)}$$

12. Method 2: Using components (Ref: N, E + ; S, W –)

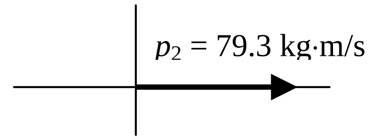
$p_1 = m_1 v_1 \;=\; (2\text{ kg})\,(72\text{ m/s}) \;=\; 144\text{ kg·m/s}$ (North)

$p_{1x} = 0$; $p_{1y} = 144\text{ kg·m/s}$



$p_2 = m_2 v_2 \;=\; (1.3\text{ kg})\,(61\text{ m/s}) \;=\; 79.3\text{ kg·m/s}$ (East)

$p_{2x} = 79.3\text{ kg·m/s}$; $p_{2y} = 0$



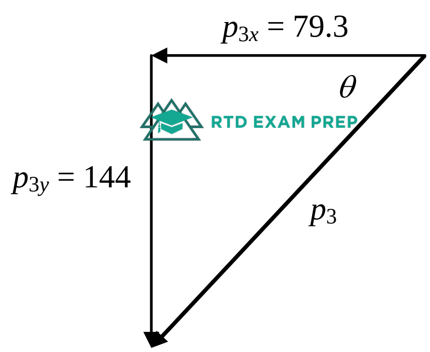
Momentum is conserved (in the x- and y- axes)

$\vec{p}_{Tx\,i} = \vec{p}_{Tx\,f}$ $\qquad 0 = \vec{p}_{1x} + \vec{p}_{2x} + \vec{p}_{3x}$ $\qquad 0 = 0 + 79.3 + \vec{p}_{3x}$

$\vec{p}_{3x} = -79.3\text{ kg·m/s}$

$\vec{p}_{Ty\,i} = \vec{p}_{Ty\,f}$ $\qquad 0 = \vec{p}_{1y} + \vec{p}_{2y} + \vec{p}_{3y}$ $\qquad 0 = 144 + 0 + \vec{p}_{3y}$

$\vec{p}_{3y} = -144\text{ kg·m/s}$



Finally, solving the triangle,

$p_3 = \sqrt{144^2 + 79.3^2} \;=\; 164\text{ kg·m/s}$

$\tan\theta = \dfrac{144}{79.3}$

$\theta = \tan^{-1}(1.816) \;=\; 61.2°$ S of W (or 28.8° W of S)

13. Momentum is conserved (in the x- and y- axes) (Ref: N, E + ; S, W –)

$\vec{p}_{Tx\,i} = \vec{p}_{Tx\,f}$ $\qquad \vec{p}_{1x\,i} + \vec{p}_{2x\,i} = \vec{p}_{1x\,f} + \vec{p}_{2x\,f}$

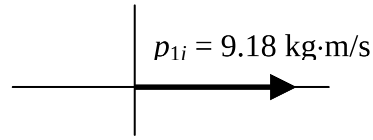
$15 + 13 = x + 20$ $\qquad x = 8\text{ kg·m/s}$

$\vec{p}_{Ty\,i} = \vec{p}_{Ty\,f}$ $\qquad \vec{p}_{1y\,i} + \vec{p}_{2y\,i} = \vec{p}_{1y\,f} + \vec{p}_{2y\,f}$

$-11 + 7 = y - 14$ $\qquad y = 10\text{ kg·m/s}$

14. **Method 1**: Components (Ref: N, E + ; S, W –)

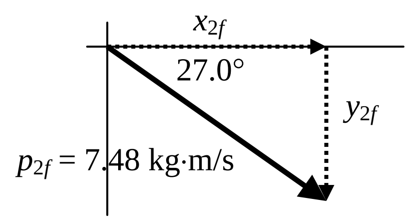
Before the collision:



$$p_{1i} = m_1 v_{1i} = (0.540 \text{ kg})(17 \text{ m/s}) = 9.18 \text{ kg·m/s}$$

$$x_{1i} = 9.18 \text{ kg·m/s} \quad ; \quad y_{1i} = 0$$

After the collision:



$$p_{2f} = m_2 v_{2f} = (0.680 \text{ kg})(11 \text{ m/s}) = 7.48 \text{ kg·m/s}$$

$$\cos 27° = \frac{x_{2f}}{7.48} \qquad x_{2f} = 7.48 \cos 27° = 6.665 \text{ kg·m/s}$$

$$\sin 27° = \frac{y_{2f}}{7.48} \qquad y_{2f} = -7.48 \sin 27° = -3.396 \text{ kg·m/s}$$

Momentum is conserved (in the x- and y- axes)

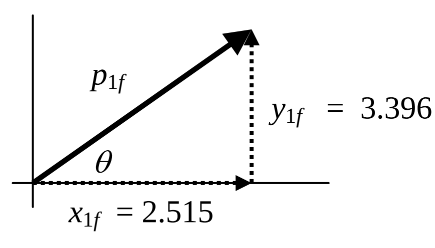
$$\vec{p}_{Tx\,i} = \vec{p}_{Tx\,f} \qquad \vec{x}_{1i} + \vec{x}_{2i} = \vec{x}_{1f} + \vec{x}_{2f}$$

$$9.18 + 0 = x_{1f} + 6.665 \qquad x_{1f} = 2.515 \text{ kg·m/s}$$

$$\vec{p}_{Ty\,i} = \vec{p}_{Ty\,f} \qquad \vec{y}_{1i} + \vec{y}_{2i} = \vec{y}_{1f} + \vec{y}_{2f}$$

$$0 + 0 = y_{1f} - 3.396 \qquad y_{1f} = 3.396 \text{ kg·m/s}$$

Solving the triangle,



$$p_{1f} = \sqrt{2.515^2 + 3.396^2} = 4.226 \text{ kg·m/s}$$

$$p = mv \qquad v = \frac{p_{1f}}{m_1} = \frac{4.226}{0.54} = 7.83 \text{ m/s}$$

$$\tan\theta = \frac{3.396}{2.515}$$

$$\theta = \tan^{-1}(1.35) = 53.5° \text{ N of E} \ \ (\text{or } 36.5° \text{ E of N})$$

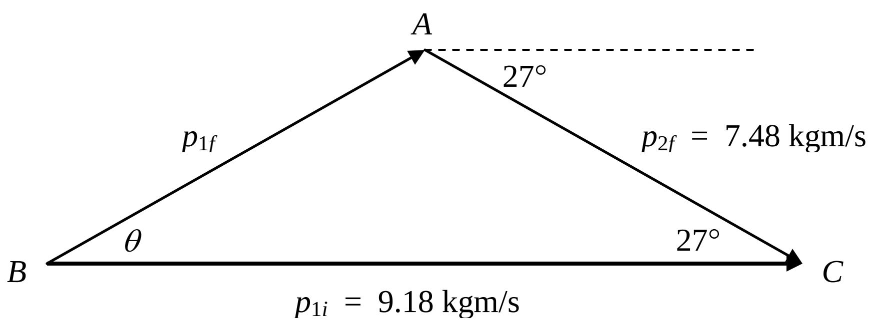
Method 2: Triangle Method

$p_{1i} = m_1 v_{1i}$ = (0.540 kg) (17 m/s) = 9.18 kg·m/s

$p_{2f} = m_2 v_{2f}$ = (0.680 kg) (11 m/s) = 7.48 kg·m/s

Conservation of momentum

$\vec{p}_{Ti} = \vec{p}_{Tf}$ $\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f}$



Using the cosine law,

$c^2 = a^2 + b^2 - 2ab\cos C$ $p_{1f}^2 = 7.48^2 + 9.18^2 - 2(7.48)(9.18)\cos 27°$

$p_{1f}^2 = 7.48^2 + 9.18^2 - 2(7.48)(9.18)\cos 27°$ $p_{1f}^2 = 17.858$

$p_{1f} = \sqrt{17.858}$ = 4.226 kg·m/s

$p = mv$ $v = \frac{p_{1f}}{m_1} = \frac{4.226}{0.54}$ = 7.83 m/s

Using the sine law,

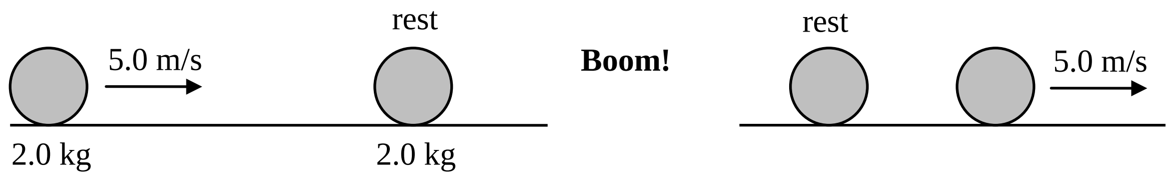
$\frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{\sin\theta}{7.48} = \frac{\sin 27°}{4.226}$ $\sin\theta = \left(\frac{\sin 27°}{4.226}\right)\cdot(7.48)$

$\sin\theta = 0.8036$ $\theta = \sin^{-1}(0.8036)$ = 53.5° N of E (or 36.5° E of N)

15. a) If it is a collision, momentum is conserved. (Assuming it is isolated)

Since they stick together, the collision cannot be elastic (in fact, it is perfectly inelastic). Thus, total kinetic energy is not conserved, since energy will be lost due to heat and sound.

b) The collision works out to be as follows:



Momentum is conserved (total is 10 kgm/s before and after).

Total kinetic energy is also conserved (total is 25 J before and after). Thus, the collision is elastic.

16. Since no angles are given, you cannot solve this problem using conservation of momentum. However, since the collision is elastic, total kinetic energy is conserved.

$$E_{kT\,i} = E_{kT\,f}$$

$$E_{k1i} = E_{k1f} + E_{k2f}$$

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1v_{1i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2$$

$$(8.0)(16)^2 = (8.0)(11)^2 + 5v^2$$

$$5v^2 = 1080$$

$$v^2 = 216$$

$$v = \sqrt{216} \quad = \quad 14.7 \text{ m/s}$$